MATH 54 - HINTS TO HOMEWORK 5

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Here are a couple of hints to Homework 5! Enjoy :)

SECTION 1.8: INTRODUCTION TO LINEAR TRANSFORMATIONS

1.8.1. Calculate $A\mathbf{u}$ and $A\mathbf{v}$.

1.8.3, 1.8.9, 1.8.11. Just solve the equation $A\mathbf{x} = \mathbf{b}$, where in $1.8.3$, $\mathbf{b}$ is given, in $1.8.9$, $\mathbf{b} = \mathbf{0}$, and in $1.8.11$, $\mathbf{b}$ is also given.

1.8.17. Use the fact that $T(3\mathbf{u}) = 3T(\mathbf{u})$, $T(2\mathbf{v}) = 2T(\mathbf{v})$ and $T(3\mathbf{u} + 2\mathbf{v}) = 3T(\mathbf{u}) + 2T(\mathbf{v})$.

1.8.21.

(a) T (it's a function with two 'linearity' properties)
(b) F ($\mathbf{R}^5$. Think in terms of variables, there are 5 variables here)
(c) F (Consider $A =$ the 0-matrix, then the range is just $\{\mathbf{0}\}$. All that we know is that the range is a *subspace* of $\mathbb{R}^m$)
(d) *technically* F (this is complicated, but for finite-dimensional vector spaces, every linear transformation is a matrix transformation, but for infinite-dimensional v.s. there are some linear transformations which cannot be represented by a matrix, like $T(y) = y'$)
(e) T (the 'only if' implication is easy, for the 'if' part, consider $c_1 = c_2 = 1$ and $c_2 = 0$)

1.8.29.

(a) Show $f(x + y) = f(x) + f(y)$ and $f(cx) = cf(x)$ where c is in $\mathbb{R}$
(b) Remember that $T(\mathbf{0}) = \mathbf{0}$ for any linear transformation!
(c) It's called linear because its graph is a line! Here the French terminology is better: They call $f(x) = mx$ a linear function (fonction lineaire), but $f(x) = mx + b$ an affine function (fonction affine)

1.8.32. Find two explicit points $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ such that $T(\mathbf{x} + \mathbf{y}) \neq T(\mathbf{x}) + T(\mathbf{y})$, **OR** find an explicit point $\mathbf{x} = (x_1, x_2)$ and an explicit constant c such that $T(c\mathbf{x}) \neq cT(\mathbf{x})$. It's important that your examples be explicit! For example, $\mathbf{x} = (0, 1)$ and $c = -2$ work! In general, things with absolute values or x^2 terms or $\sin(x)$ terms will not be linear!

1.8.33. Remember that $T(\mathbf{0}) = \mathbf{0}$

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Section 1.9: The matrix of a linear transformation

For **all** of those questions, all you need to find is $T(\mathbf{e_1}), T(\mathbf{e_2})$, and sometimes $T(\mathbf{e_3})$, where $\mathbf{e_1} = \begin{bmatrix}1\\0\end{bmatrix}$ or $\begin{bmatrix}1\\0\\0\end{bmatrix}$, etc. and put all the vectors your found in a matrix!

1.9.11. Calculate the matrix of T. The angle of rotation is π radians (180 degrees).

1.9.23.

(a) **T** (in other words, if you know $T(\mathbf{e_1}), T(\mathbf{e_2}), \cdots, T(\mathbf{e_n})$, you know T)
(b) **T** (see example 3)
(c) **F** (the composition of two linear transformations is a linear transformation, see chapter 4)
(d) **F** (onto means every vector in $\mathbb{R}^n$ is in the image of T)
(e) **F** (let $A = \begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}$, then the columns of A are linearly independent, and hence T is one-to-one by theorem $12b$)

1.9.36. The hint the book gives is a bit complicated! It's easier to calculate $T(S(\mathbf{u}+\mathbf{v}))$ and $T(S(c\mathbf{u}))$. First use the fact that S is linear, then use the fact that T is linear!

Section 5.4: Eigenvectors and linear transformations

For *all* the questions in the section, you need to do the following 3 things:

1) For every element $\mathbf{b_i}$ in the basis, calculate $T(\mathbf{b_i})$
2) Calculate $[T(\mathbf{b_i})]_{\mathcal{C}}$, i.e. the code of $T(\mathbf{b_i})$ with respect to the output-basis $\mathcal{C}$
3) Put all the vectors you found in a matrix!

5.4.1. The matrix in question is $A = \begin{bmatrix}[\mathbf{T}(\mathbf{b_1})]_{\mathcal{D}} & [\mathbf{T}(\mathbf{b_2})]_{\mathcal{D}} & [\mathbf{T}(\mathbf{b_3})]_{\mathcal{D}}\end{bmatrix}$

5.4.2. The matrix in question is $A = \begin{bmatrix}[\mathbf{T}(\mathbf{d_1})]_{\mathcal{B}} & [\mathbf{T}(\mathbf{d_2})]_{\mathcal{C}}\end{bmatrix}$

5.4.3. Remember how to do the problem! The other problems are similar! Also, remember that $\mathbf{e_1} = \begin{bmatrix}1\\0\\0\end{bmatrix} = (1,0,0)$ etc.

5.4.5.

(a) $T(p) = (t+5)(2-t+t^2) = 10 - 3t + 4t^2 + t^3$

(b) Show $T(p+q) = T(p) + T(q)$ and $T(cp) = cT(p)$ where c is a constant

(c) Calculate $T(1)$, $T(t^2)$, $T(t^3)$ and find the 'code' of each of the polynomials you found! Careful about the order!

5.4.7. Calculate $T(1)$, $T(t)$, $T(t^2)$ and find the 'code' of each polynomial you found! Again, careful about the order! For example, the code of $T(1) = 3 + 5t$ is $\begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$, and not $\begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$

5.4.9. For (b) Show $T(\mathbf{p} + \mathbf{q}) = T(\mathbf{p}) + T(\mathbf{q})$ and $T(c\mathbf{p}) = cT(\mathbf{p})$, for (c), calculate $T(1), T(t)$ and $T(t^2)$. For (c), this is just like $5.4.7$, except finding the code is much easier!

5.4.11. $A = \begin{bmatrix} [\mathbf{T}(\mathbf{b_1})]_{\mathcal{B}} & [\mathbf{T}(\mathbf{b_2})]_{\mathcal{B}} \end{bmatrix}$. That is, calculate $T(\mathbf{b_1})$ and $T(\mathbf{b_2})$ and express your answer in terms of $\mathbf{b_1}$ and $\mathbf{b_2}$.